Black hole spectroscopy with LISA

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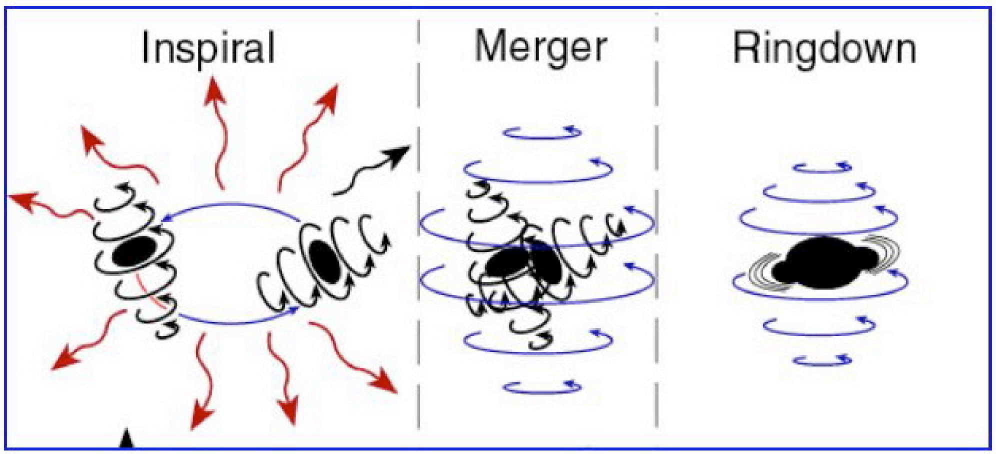
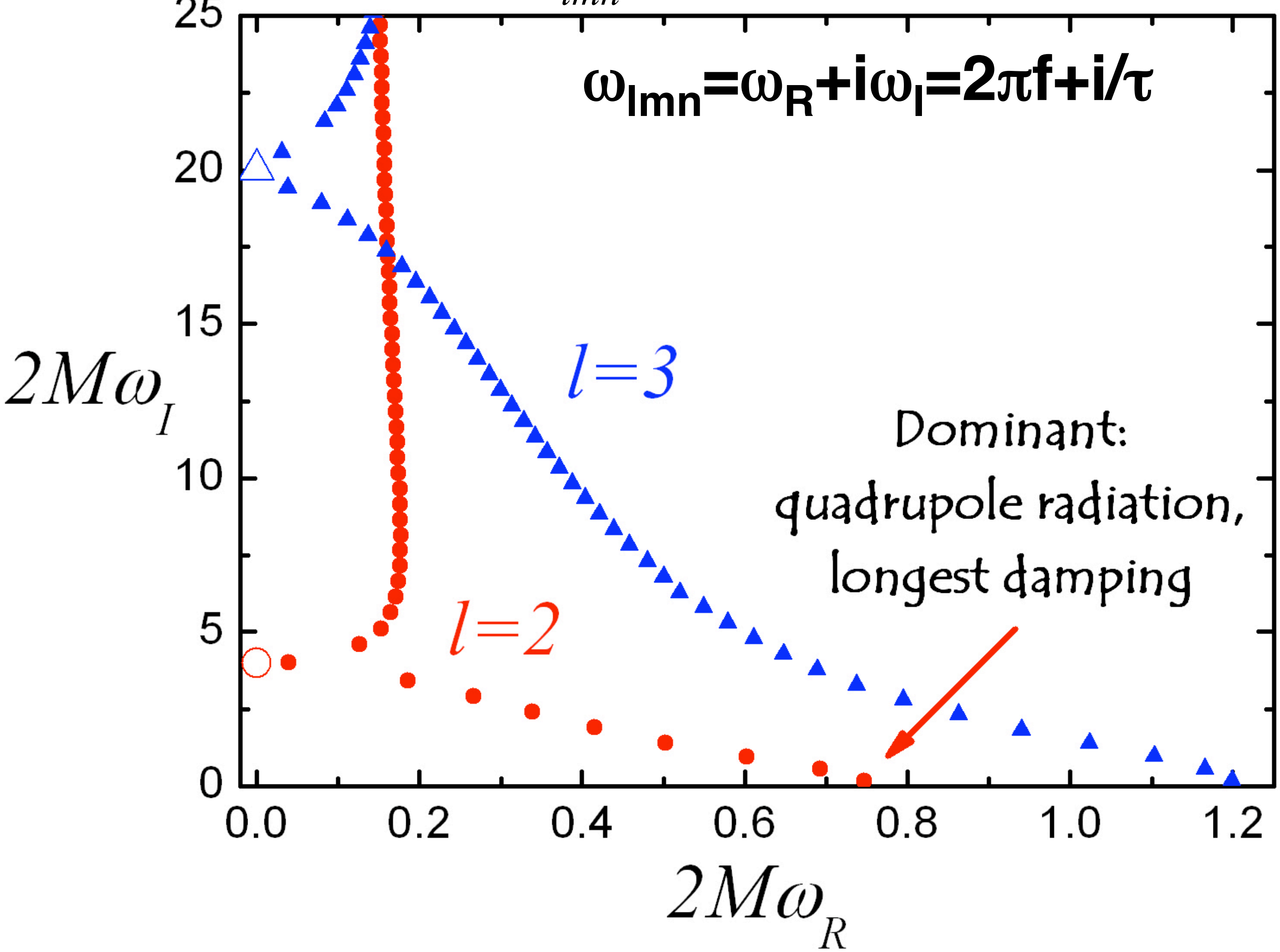


EB, Buonanno & Will: GR tests from inspiral
EB, Cardoso & Will: GR tests from ringdown (this talk)

Why "black hole spectroscopy"?

$$r\left(h_+ + ih_\times\right) = \sum_{lmn} A_{lmn} \exp(i\omega_{lmn} t) S_{lmn}(\theta,\varphi)$$



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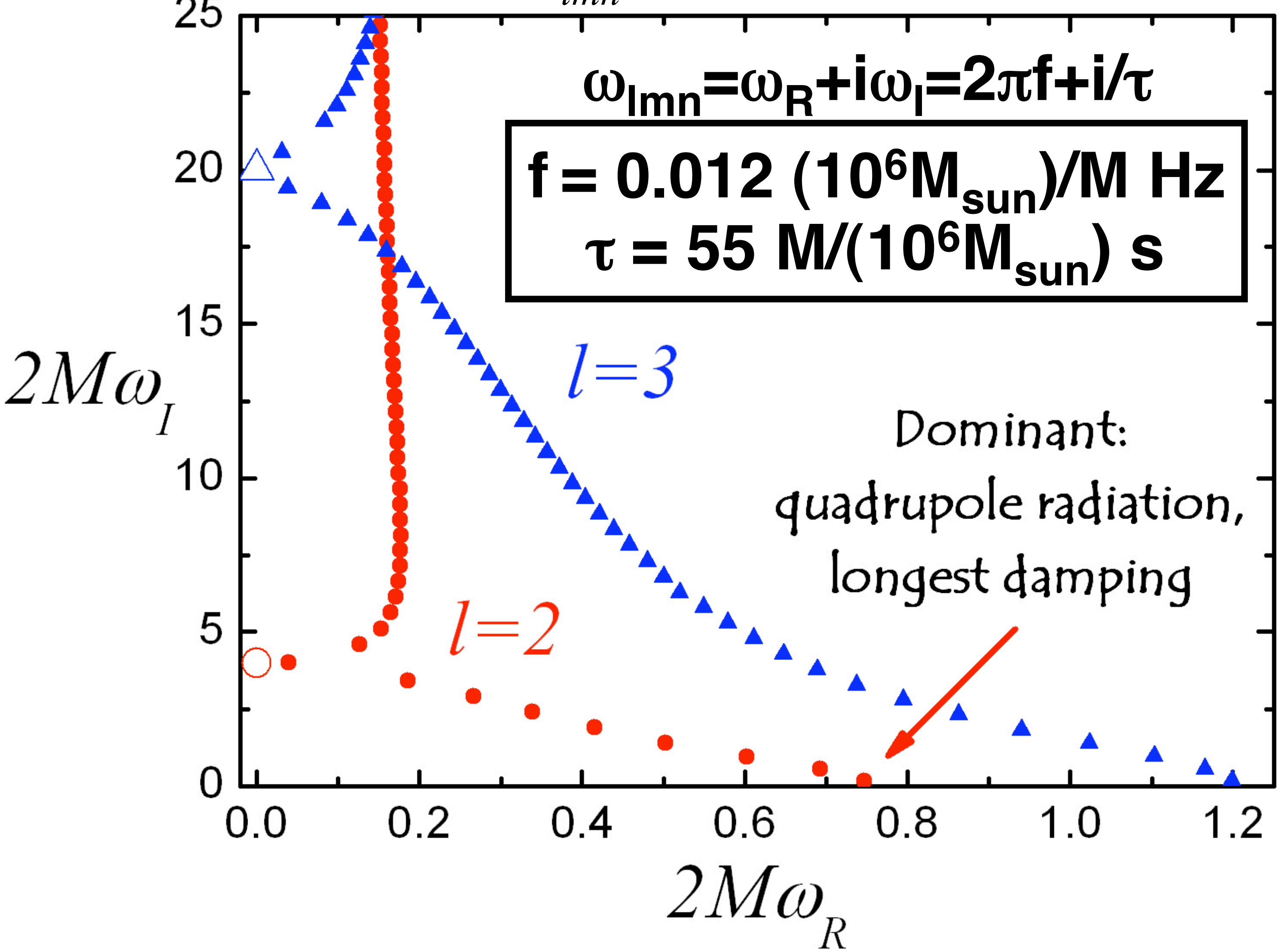
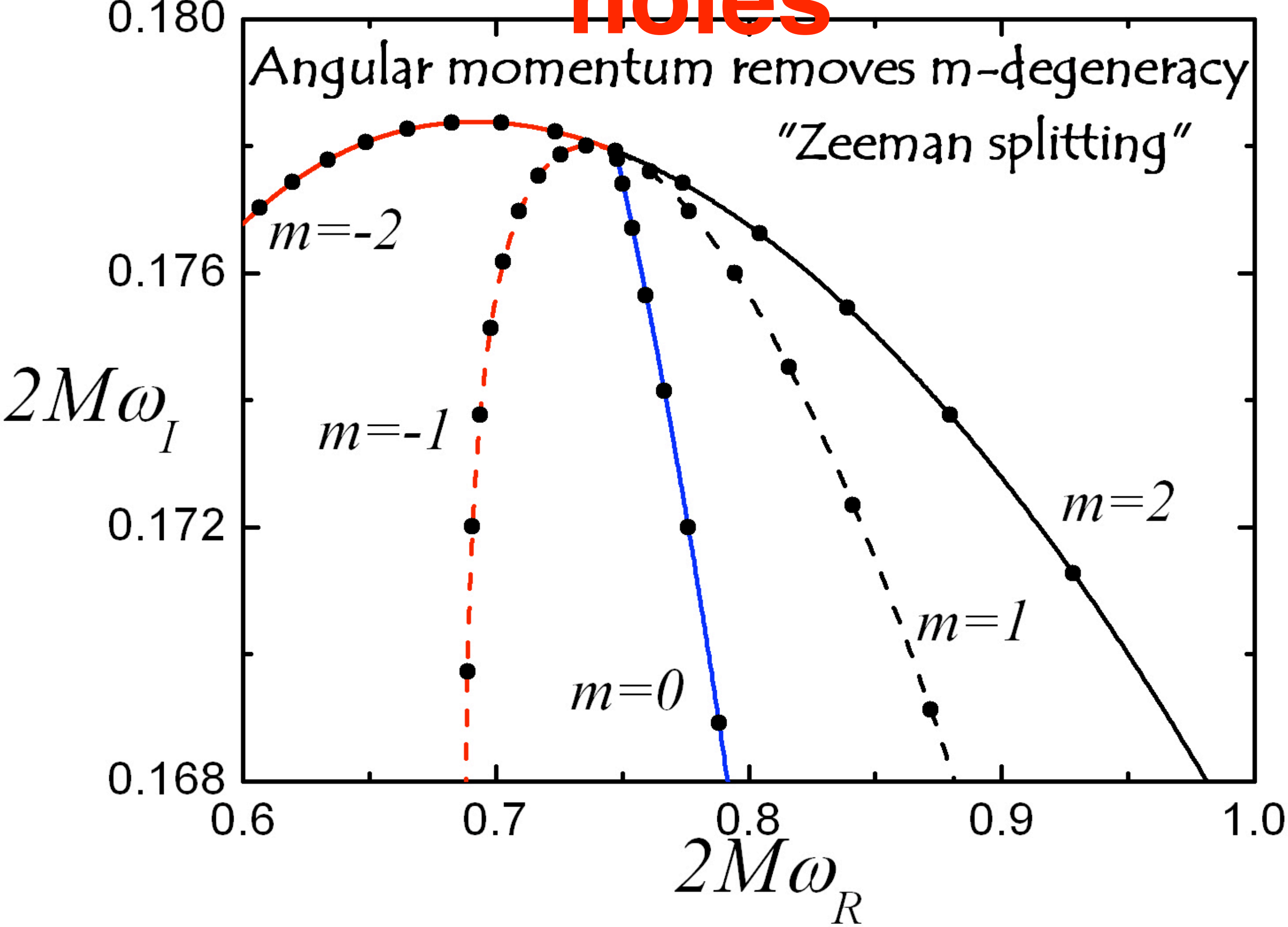


Spectroscopy of rotating black holes



Modes always come in pairs: reflection symmetry

$m \rightarrow -m \quad \omega_R \rightarrow -\omega_R$

GR tests from ringdown waves

One-mode detection:

if we know which mode we are detecting (eg. **l=m=2**)
measure of black hole's mass and angular momentum

$$\mathbf{f(M,j),\ \tau(M,j)} \longrightarrow \mathbf{M(f,\tau),\ j(f,\tau)}$$

(Echeverria, Finn)

Multi-mode detection:

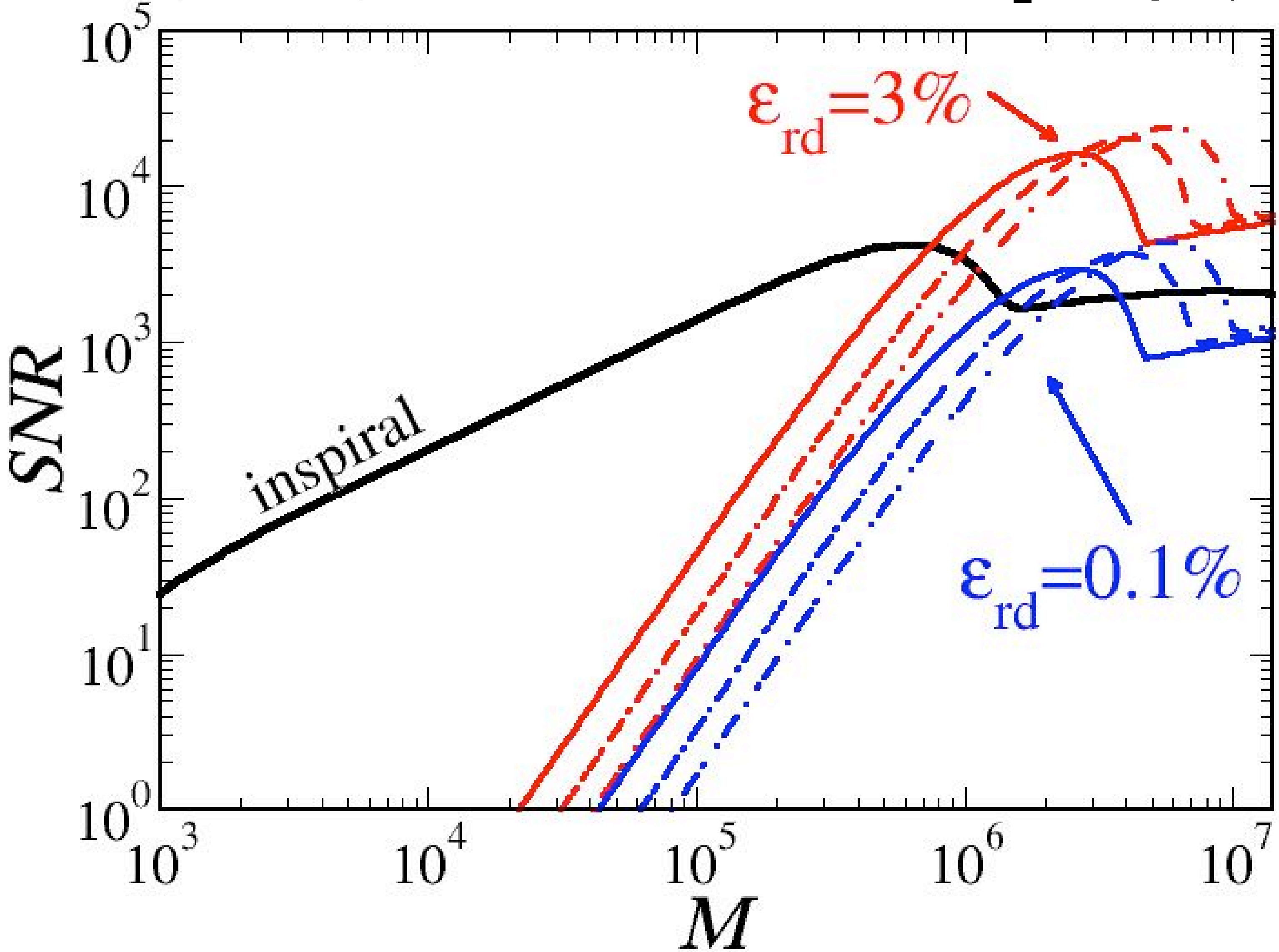
First mode yields **(M,j)**

In GR Kerr quasinormal frequencies depend ***only*** on **M** and j:
second mode yields **test** that we are observing a Kerr black hole
(Dreyer *et al.*; EB, Cardoso & Will)

Test similar in nature to "multipolar mapping" with EMRIs

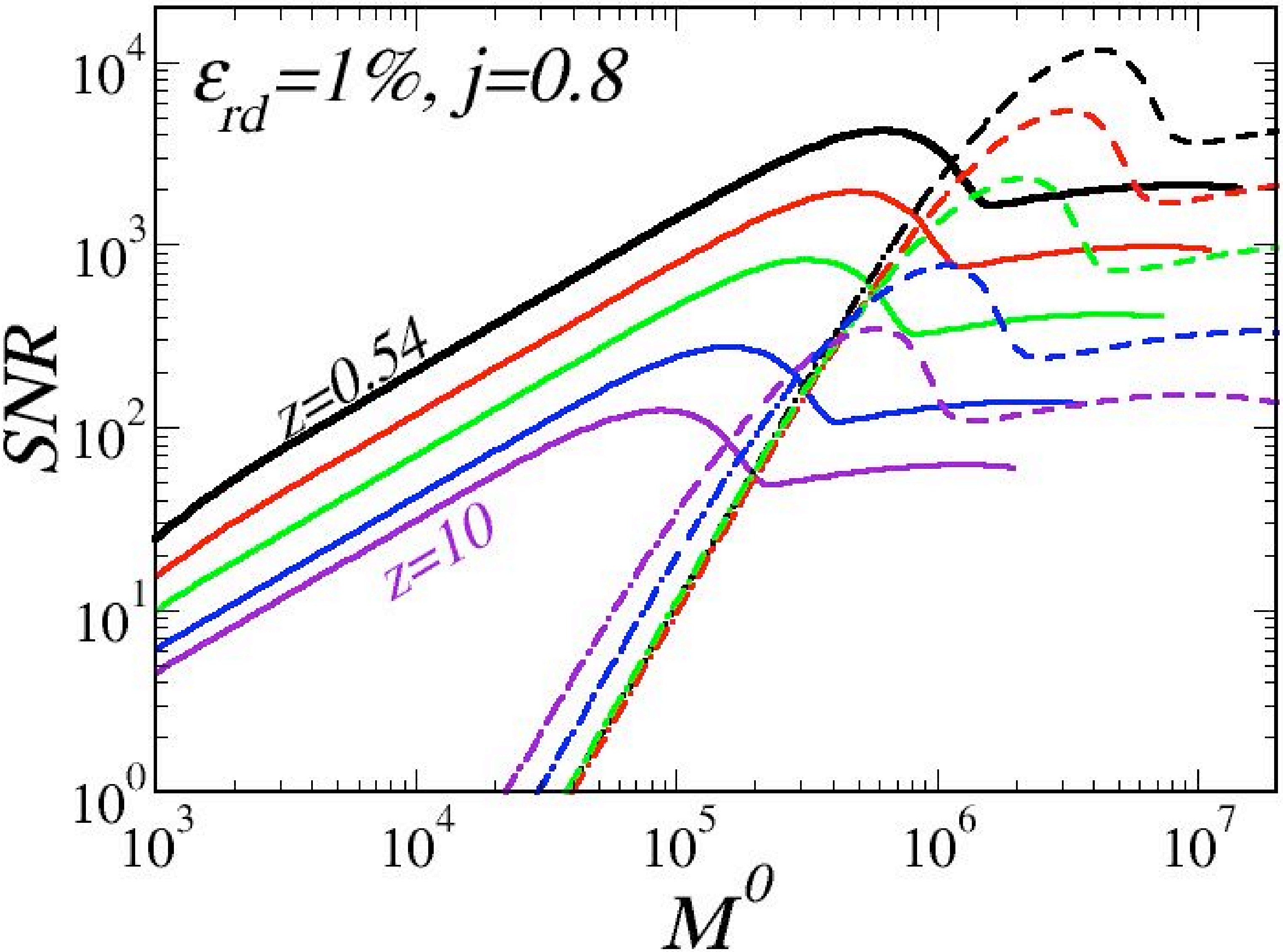
SNR for inspiral and ringdown

Inspiral of equal mass BH-BH binaries at @ **D_L=3 Gpc** (**z=0.54**)



SNR for inspiral and ringdown

Redshift dependence



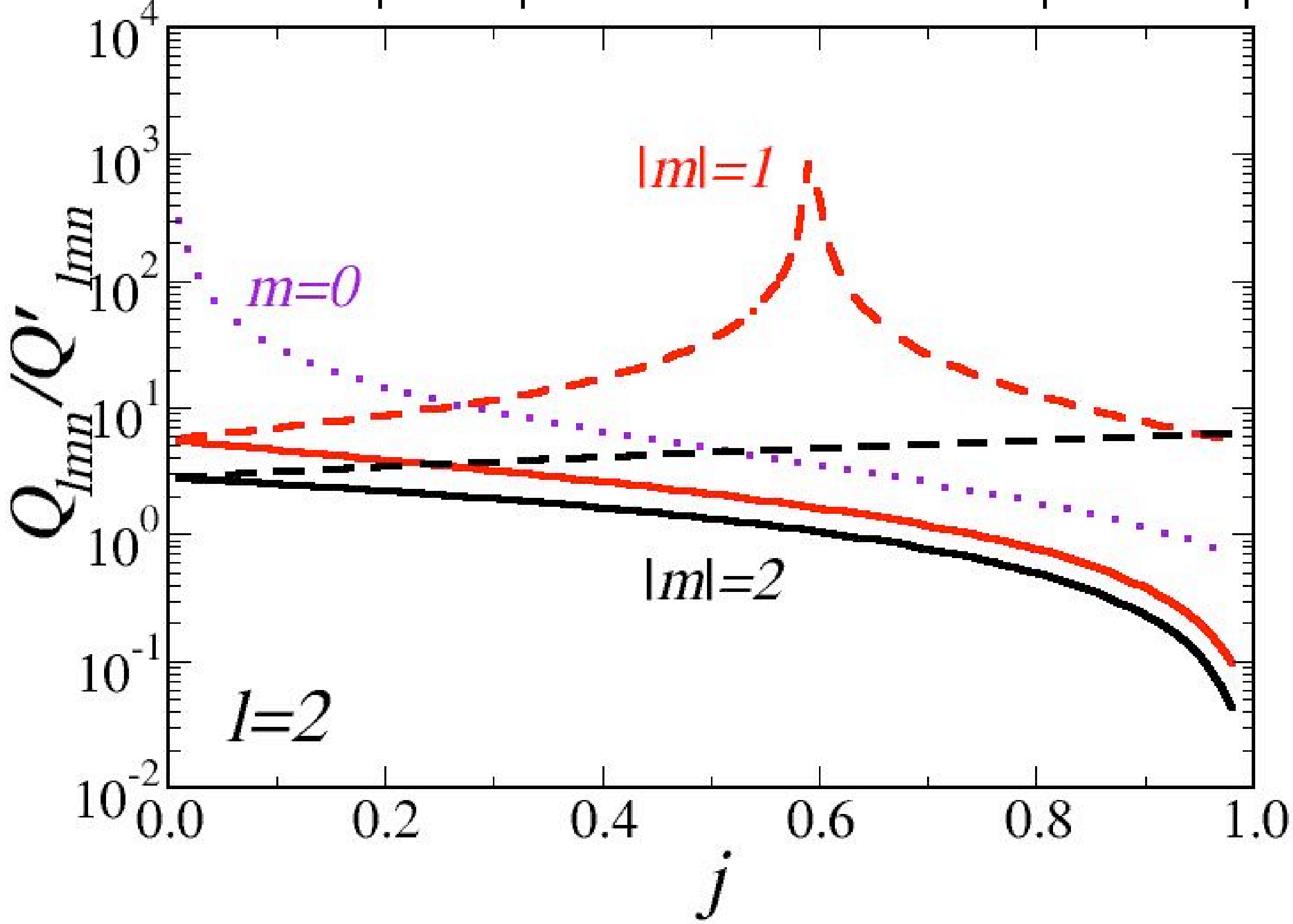
~10000
@
z=0.54

~300
@ z=10

Measurement errors on a single QNM

Errors depend on the quality factor **$Q=\pi f\tau$** and scale as **$\rho^{-1}\sim\varepsilon_{rd}^{-1/2}$**

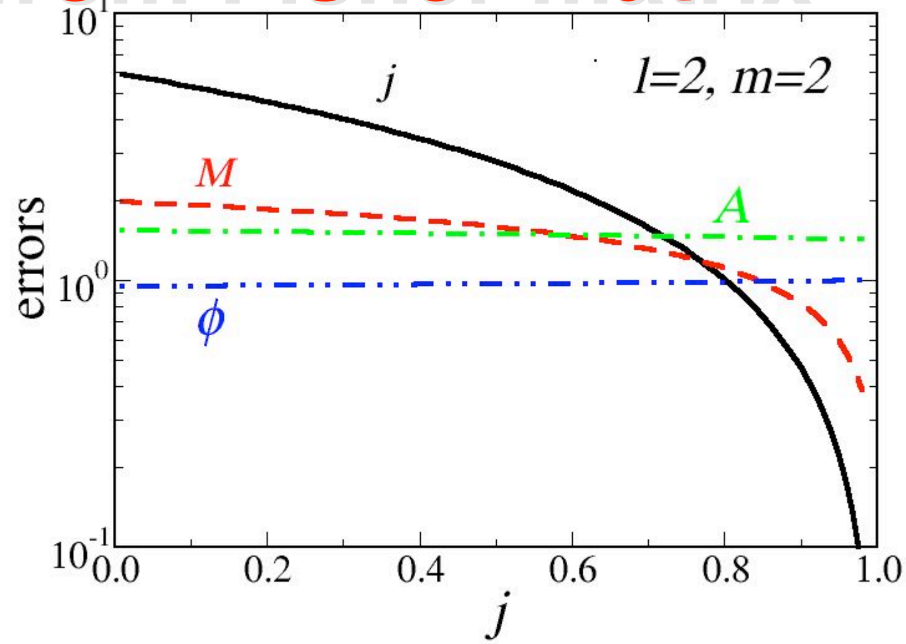
$$\rho\Delta j \approx \left|\frac{2Q}{Q'}\right| \qquad \rho\frac{\Delta M}{M} \approx \left|\frac{2Qf'}{fQ'}\right|$$

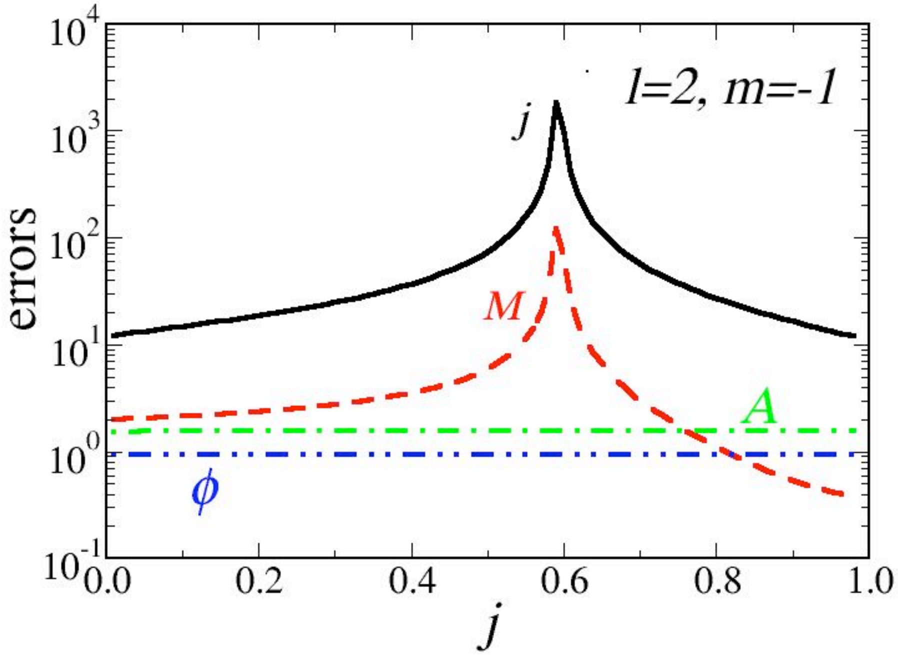


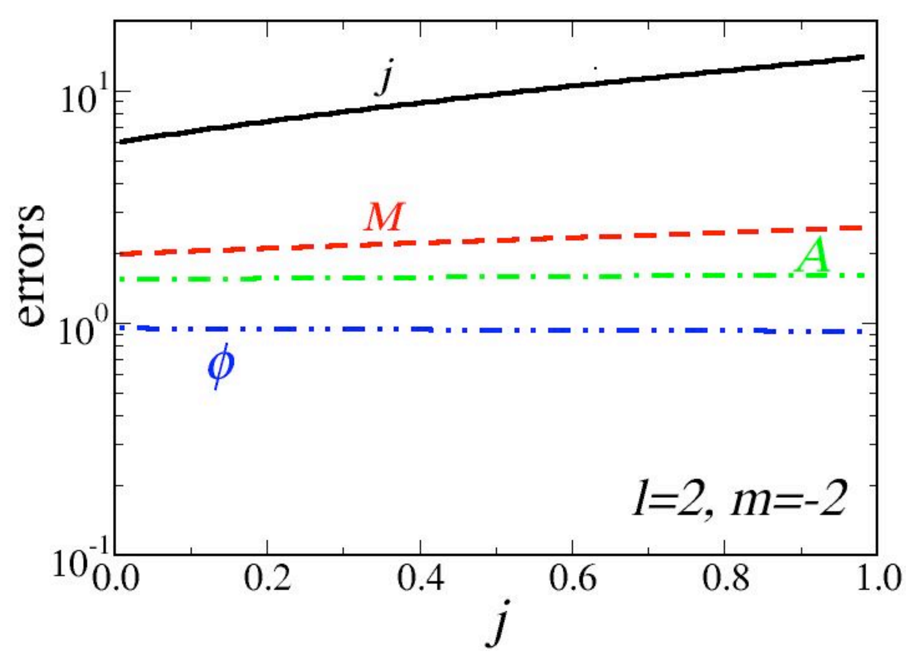
Numerical errors from Fisher matrix

$\rho\Delta j$, $\rho\Delta M/M$, $\rho\Delta A/A$, $\rho\Delta\phi$

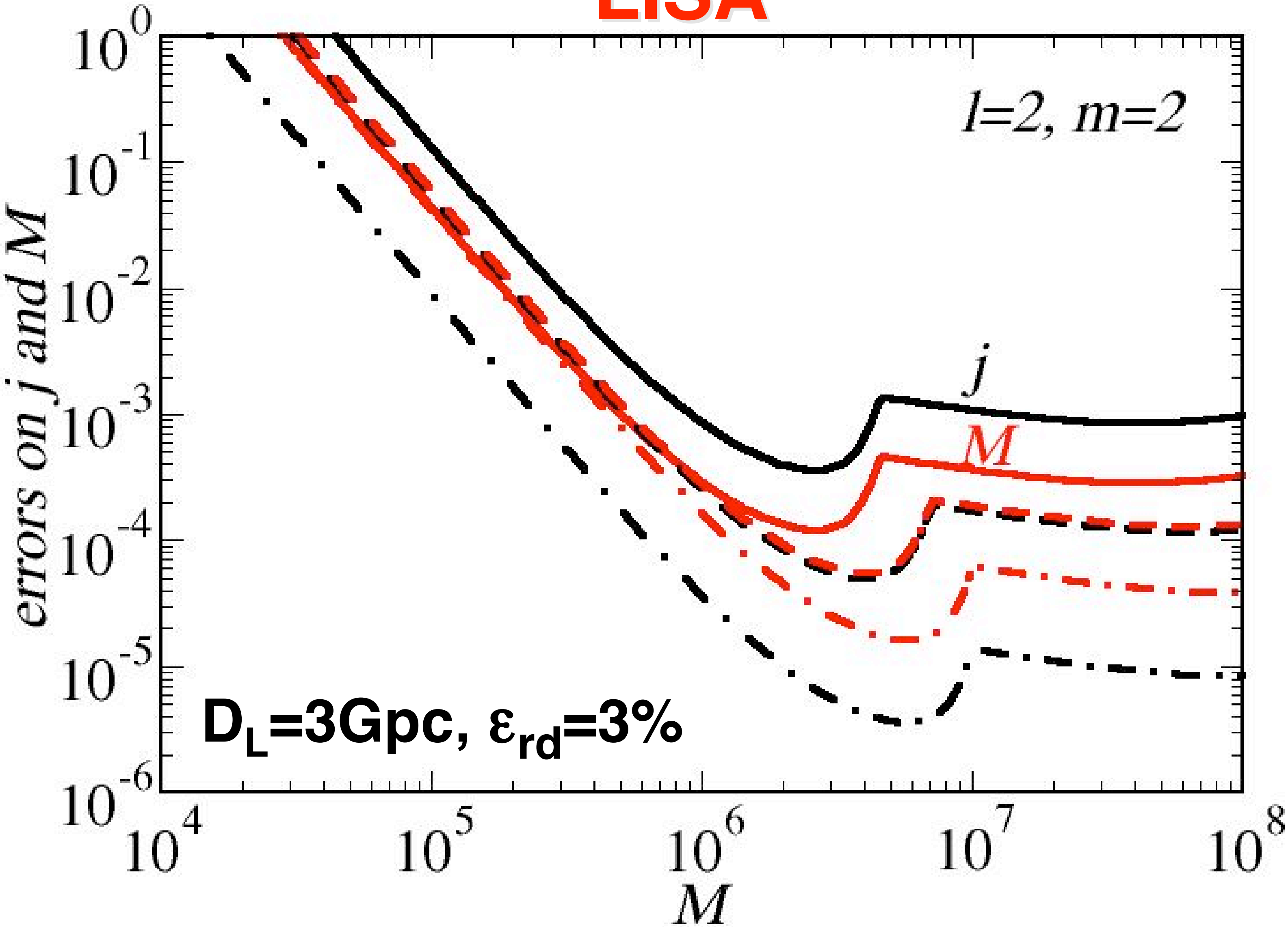
Detector-independent results: rescale by SNR for LISA, LIGO, Virgo







Errors on single-mode detection with LISA



Recall errors scale as $\rho^{-1} \sim \varepsilon_{rd}^{-1/2}$

Multi-mode ringdown detection

$$r\left(h_{+}+ih_{\times}\right)=\sum_{lmn}A_{lmn}\exp(i\omega_{lmn}t)S_{lmn}(\theta,\varphi)$$

Relative excitation?

Spin-weighted spheroidal harmonics are such that $\boxed{\int S^{*}_{lmn}S_{l'm'n'}d\Omega\approx\delta_{ll'}\delta_{mm'}}$

1. "Orthogonal" waveforms (different angular dependence):
   need ***numerical relativity*** to determine "deformation" of the hole
2. "Parallel" waveforms (same angular dependence, different overtones):
   numerical relativity initial data + perturbative "excitation factors" $\mathbf{B_{lmn}}$

$$\psi_{lmn}(t,r)=-\Re\sum_{lmn}\left\{B_{lmn}\left(\int_{-\infty}^{\infty}I(\omega,r)\hat{\psi}_{lmn}dr_{*}'\right)\times\exp[-i\omega_{lmn}(t-r_{*})]\right\}$$

Computing $\mathbf{B_{lmn}}$ suggests overtones are more important for fast rotation

QNM resolvability

Tests of the no-hair theorem need a measurement of at least TWO QNMs
No-go theorem: which SNR do we need to resolve two QNMs?

Rayleigh-like criterion:

$$|f_1 - f_2| > \max(\sigma_{f_1}, \sigma_{f_2}) \qquad |\tau_1 - \tau_2| > \max(\sigma_{\tau_1}, \sigma_{\tau_2})$$

Detector-independent critical SNR:

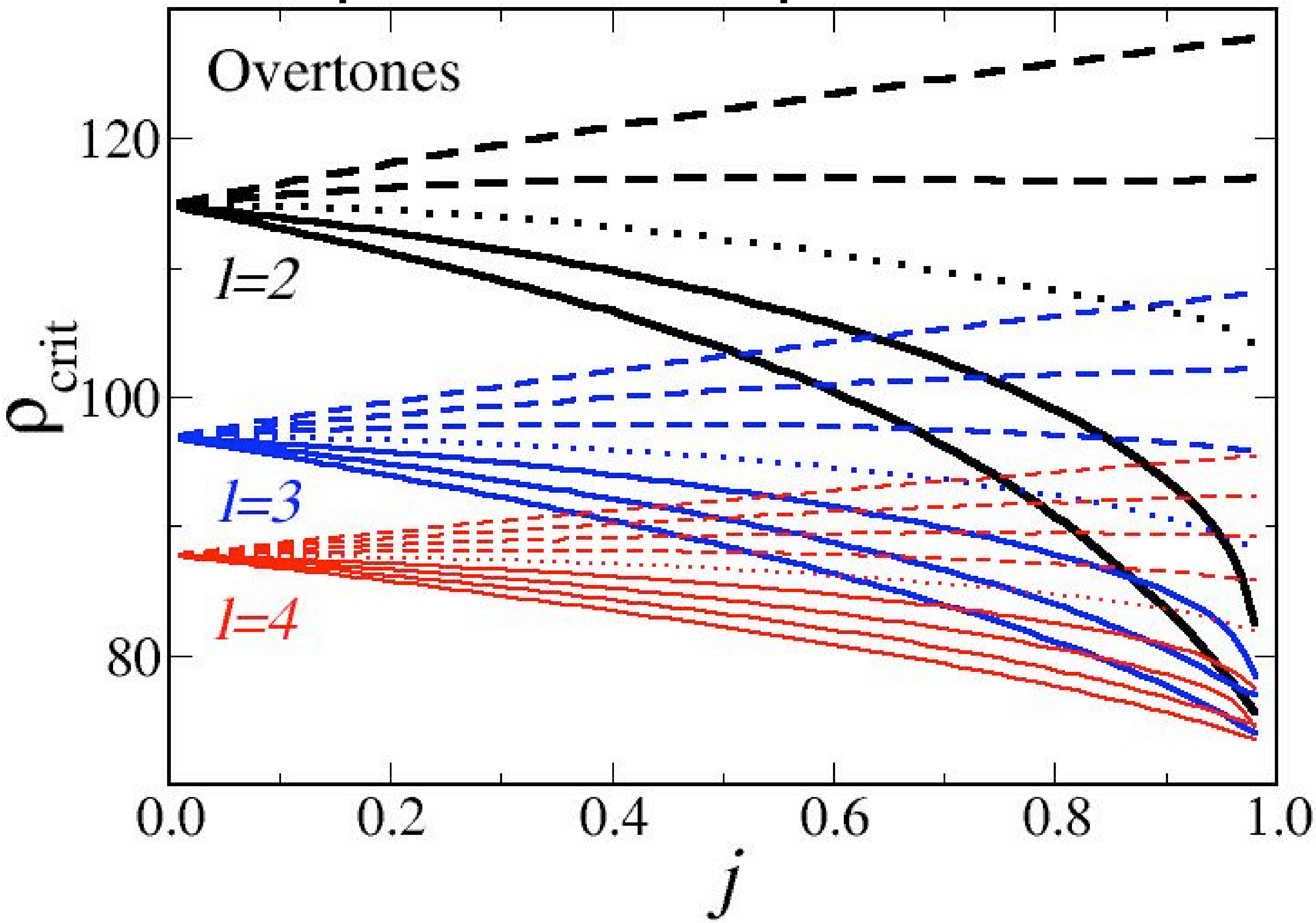
$$\rho^{f}_{crit} > \frac{\max(\rho\sigma_{f_1}, \rho\sigma_{f_2})}{|f_1 - f_2|} \qquad \rho^{\tau}_{crit} > \frac{\max(\rho\sigma_{\tau_1}, \rho\sigma_{\tau_2})}{|\tau_1 - \tau_2|}$$

Different results for “orthogonal” and “parallel” waveforms

Critical SNR for resolvability

"Reasonable" assumption: 1% of ringdown energy in the second mode

LISA: **ρ~10^4 @ z=0.5, ρ~300 @ z=10**



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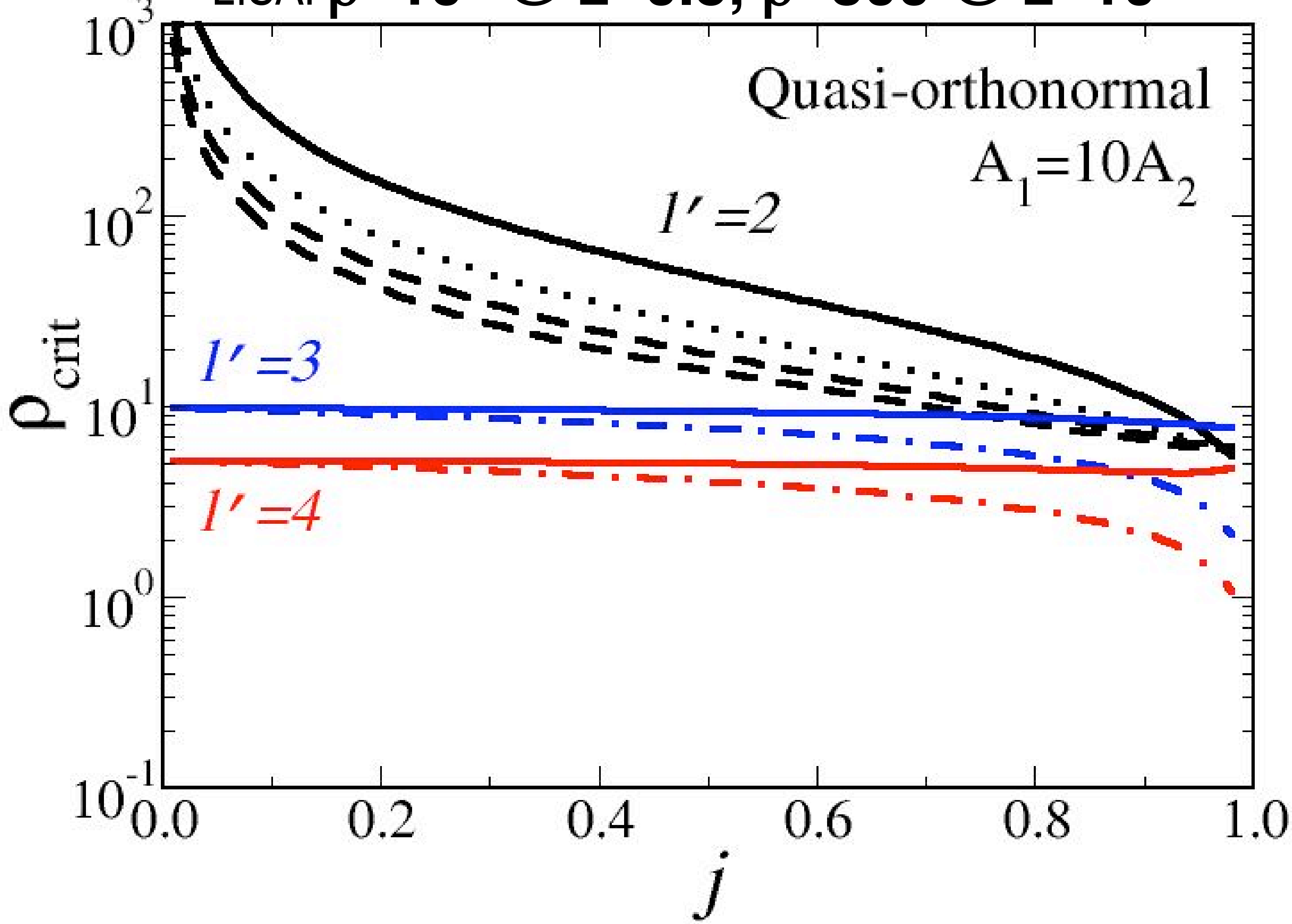
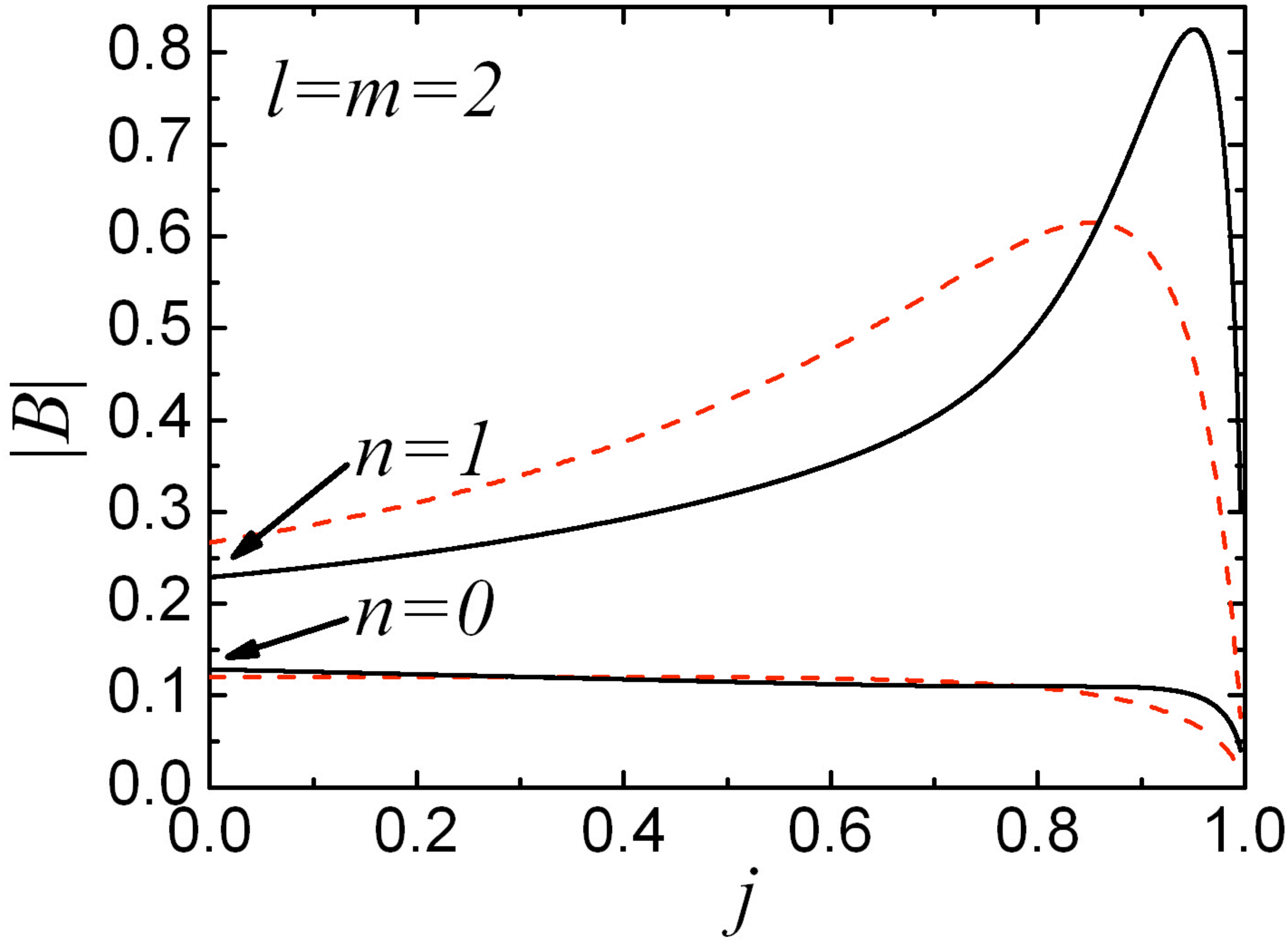


Summary

1. LISA should detect ringdown waves with **large SNR** even at large redshift **($\rho \sim 10^4$ @ z=0.5, $\rho \sim 300$ @ z=10).**
2. LISA's "sweet spot" @ **10^{-2}Hz** is **ideal for typical SMBHs** (**$\sim 10^6$ M_{sun}**). Signal duration $\tau \sim 1$ min means "simple" data analysis.
3. Very **small errors** on **M** and **j** from single-mode detections (**$\sim 10^{-6}$-10^{-3}**). Errors could decrease combining inspiral, merger and ringdown.
4. Under reasonable assumptions (to be checked by numerical relativity!) **no-hair theorem tests only require $\rho \sim 10^2$** - feasible out to large **z**.
5. Exotica:
   - No-hair tests for **IMBHs using advanced LIGO**
     Main issue: rates? (Fregeau *et al.*)
   - What if it's not a Kerr black hole?
     "Hair counting" (Ryan's "three-hair theorem" for **boson stars**)

Excitation factors for Kerr black holes



What if it's not consistent with Kerr?

Rotating boson stars (with λ): Ryan's "three-hair" theorem

Boson star collisions and evolutions:

not only a numerical exercise, plausible "strawmen"

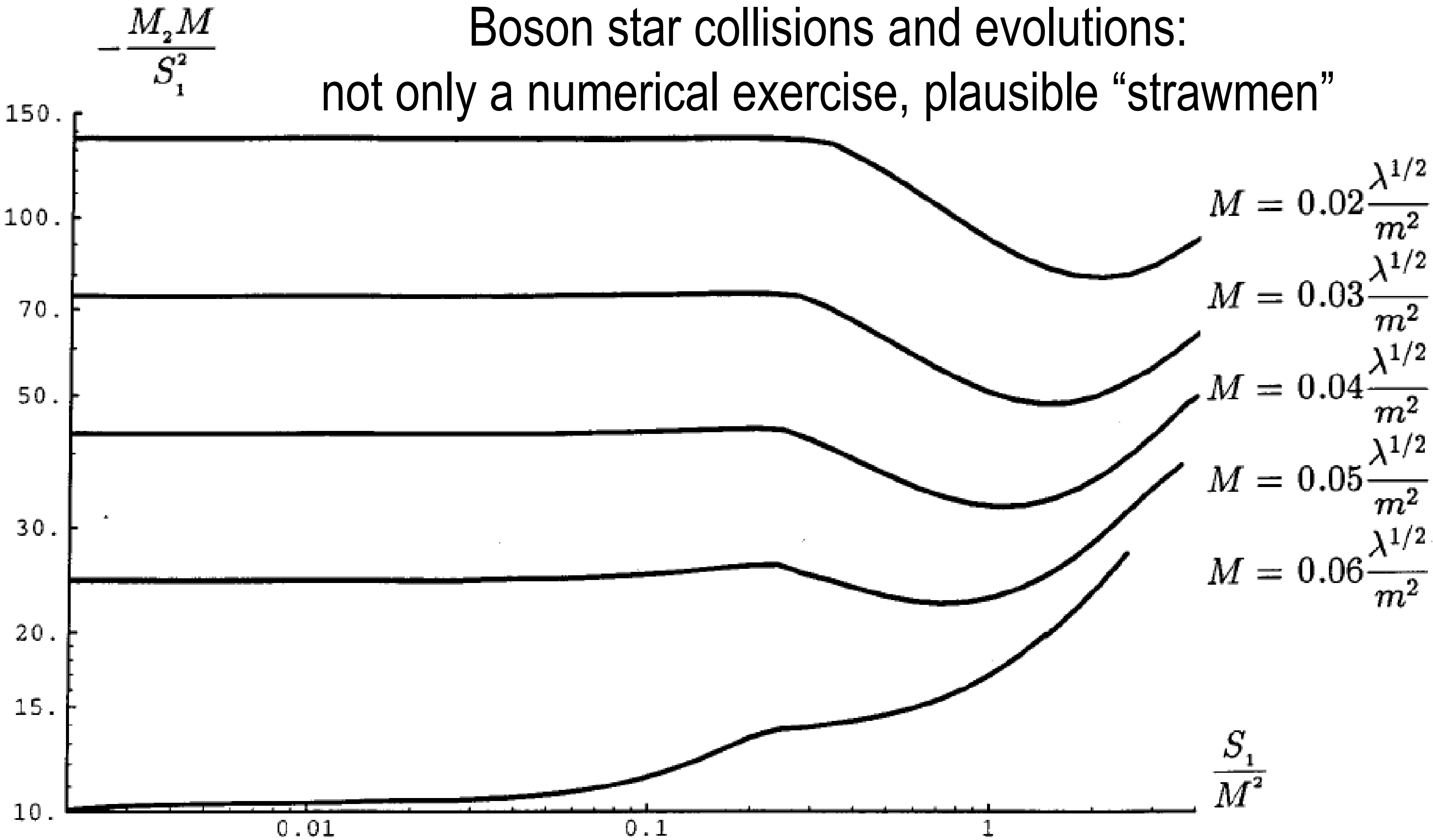


FIG. 4. A graph showing the mass quadrupole moment of a boson star as a function of the star's mass M and spin S_1. The horizontal axis is S_1/M^2 while the vertical axis is $-M_2M/S_1^2$. From top to bottom, the curves are for boson star masses of 0.02, 0.03, 0.04, 0.05, and 0.06, all in units of $\lambda^{1/2}/m^2$. Note that for a black hole $-M_2M/S_1^2=1$.